Doubly heavy baryons in sum rules of NRQCD

V.V. Kiselev[a)], A.I. Onishchenko[b)]

[a)] *State Research Center of Russia "Institute for High Energy Physics", Protvino, Moscow region, 142284 Russia*

[b)] *Institute for Theoretical and Experimental Physics, Moscow, 117218 Russia.*

Abstract

The masses of baryons containing two heavy quarks and their couplings to the corresponding quark currents are evaluated in the framework of NRQCD sum rules. The coulomb-like corrections in the system of doubly heavy diquark are taken into account, and the contribution of nonperturbative terms coming from the quark, gluon and mixed condensates as well as the product of quark and gluon condensates, is analyzed. The higher condensates destroy the factorization of baryon and diquark correlators and provide the convergency of sum rule method. As a result the accuracy of estimates is improved.

1 Introduction

Along with an experimental search for an explanation of electroweak symmetry breaking and a physics beyond the Standard Model, high energies and luminosities of particle accelerators running or being planned and under construction provide a possibility to observe rare processes with heavy quarks. A competitive topic here is a study of dynamics in flavored hadrons containing two heavy quarks. It can play a fundamental role in an extraction of primary parameters of quark interactions, since a distinction between the QCD effects inside the doubly and singly heavy hadrons allows one strictly to constrain incalculable nonperturbative quantities determining the isolation of pure electroweak physics.

The real possibility of such experimental measurements was recently confirmed by CDF Collaboration due to the first observation of B_c meson [1]. As predicted theoretically [2], this long-lived state of $\bar{b}$ and c quarks has the production cross sections, mass and decay rates, which represent characteristic values for the doubly heavy hadrons. Thus, the experimental search for the doubly heavy baryons can also be successful. Of course, such the search would be more strongly motivated if it would be supported by modern theoretical studies and evaluations of basic characteristics for the doubly heavy baryons.

Some steps forward this program were already done. First, the production cross sections of doubly heavy baryons in hadron collisions at high energies of colliders and in fixed target experiments were calculated in the framework of perturbative QCD for the hard processes and factorization of soft term related to the nonperturbative binding the heavy quarks [3]. Second, the lifetimes and branching fractions of basic decay modes were evaluated in the Operator Product Expansion combined with the effective theory of heavy quarks, which results in series over the inverse heavy quark masses and relative velocities of heavy quarks inside the doubly heavy diquark [4]. Third, the families of doubly heavy baryons, which contain a set of narrow excited levels in addition to the basic state, were described in the framework of potential models [5], so that the picture of spectra is very similar to that of heavy quarkonia. Fourth, the QCD sum rules [6] were explored for the two-point baryonic currents in order to calculate the masses and couplings of doubly heavy baryons [7]. However, the latter analysis contains some disadvantages, which are related to instability of sum rules in a region of parameters defining the baryonic currents. This fact leads to quite large uncertainties in the calculations.

In the present paper we analyze the NRQCD sum rules for the two-point correlators of currents corresponding to the doubly heavy baryons. The basic physical motivation of such consideration is a nonrelativistic motion of two heavy quarks inside a small size diquark bound with a light quark. This fact leads to the definite expressions for the structure of baryonic currents written down in terms of nonrelativistic heavy quarks. To the leading order of inverse heavy quark mass and relative velocity of heavy quarks inside the diquark, the NRQCD sum rules require the account for hard gluon corrections to relate the nonrelativistic heavy quark correlators to the full QCD ones. The corresponding anomalous dimensions of baryonic currents were calculated with the two-loop accuracy [8]. The NRQCD structure of currents corresponds to a definite choice of parameters in the full QCD expressions. Those values of parameters are inside the instability region, observed in the analysis done previously [7]. We find the simple physical reason for this loose of convergency: the behaviour of quantities versus the sum rule parameters (the Borel variable or the moment number of spectral density) is determined by the presence of doubly heavy diquark inside the baryon and, hence, the difference

between the masses of baryon and diquark. This mass difference takes a dominant role unless we introduce the corrections corresponding to the essential nonperturbative interactions between the doubly heavy diquark and the light quark composing the baryon. In NRQCD sum rules this introduction is realized in terms of nonperturbative condensates caused by higher dimension operators. We show that the stability of sum rules can be reached due to the account for the product of quark and gluon condensates in addition to the quark, gluon and mixed condensates. This product was not taken into account in the previous analysis in full QCD. Moreover, we carefully take into account the coulomb-like α_s/v corrections inside the heavy diquark, which enhances the relative weight of perturbative parts with respect to the condensate terms in the calculated correlators.

In Section 2 we define the currents and represent the spectral densities in the NRQCD sum rules for various operators included into the consideration. Section 3 is devoted to the numerical estimates. We find the masses of basic states, which are close to the values obtained in the potential models. The results are summarized in Conclusion.

2 NRQCD sum rules for doubly heavy baryons

2.1 Baryonic Currents

The currents of baryons with two heavy quarks $\Xi_{cc}^{\diamond}$, $\Xi_{bb}^{\diamond}$ and $\Xi_{bc}^{\prime\diamond}$, where $\diamond$ denotes various electric charges depending on the flavour of light quark, are associated with the spin-parity quantum numbers $j_d^P = 1^+$ and $j_d^P = 0^+$ for the heavy diquark system with the symmetric and antisymmetric flavor structure, respectively. Adding the light quark to the heavy quark system, one obtains $j^P = \frac{1}{2}^+$ for the $\Xi_{bc}^{\prime\diamond}$ baryons and the pair of degenerate states $j^P = \frac{1}{2}^+$ and $j^P = \frac{3}{2}^+$ for the baryons $\Xi_{cc}^{\diamond}$, $\Xi_{bc}^{\diamond}$, $\Xi_{bb}^{\diamond}$ and $\Xi_{cc}^{*\diamond}$, $\Xi_{bc}^{*\diamond}$, $\Xi_{bb}^{*\diamond}$. The structure of baryonic currents with two heavy quarks is generally chosen as

$$J = [Q^{iT} C\Gamma\tau Q^j]\Gamma' q^k \varepsilon_{ijk}. \tag{1}$$

Here T means transposition, C is the charge conjugation matrix with the properties $C\gamma_\mu^T C^{-1} = -\gamma_\mu$ and $C\gamma_5^T C^{-1} = \gamma_5$, i, j, k are colour indices and τ is a matrix in the flavor space. The effective static field of the heavy quark is denoted by Q. To the leading order over both the relative velocity of heavy quarks and their inverse masses, this field contains the "large" component only in the hadron rest frame.

Here, unlike the case of baryons with a single heavy quark [9], there is the only independent current component J for each of the ground state baryon currents. They equal

$$\begin{aligned} J_{\Xi_{QQ'}^{\prime\diamond}} &= [Q^{iT} C\tau\gamma_5 Q^{j\prime}] q^k \varepsilon_{ijk}, \\ J_{\Xi_{QQ}^{\diamond}} &= [Q^{iT} C\tau\boldsymbol{\gamma}^m Q^j]\cdot \boldsymbol{\gamma}_m \gamma_5 q^k \varepsilon_{ijk}, \\ J^n_{\Xi_{QQ}^{*\diamond}} &= [Q^{iT} C\tau\boldsymbol{\gamma}^n Q^j] q^k \varepsilon_{ijk} + \frac{1}{3}\boldsymbol{\gamma}^n [Q^{iT} C\boldsymbol{\gamma}^m Q^j]\cdot \boldsymbol{\gamma}_m q^k \varepsilon_{ijk}, \end{aligned} \tag{2}$$

where $J^n_{\Xi_{QQ}^{*\diamond}}$ satisfies the spin-3/2 condition $\boldsymbol{\gamma}_n J^n_{\Xi_{QQ}^{*\diamond}} = 0$. The flavor matrix τ is antisymmetric for $\Xi_{bc}^{\prime\diamond}$ and symmetric for $\Xi_{QQ}^{\diamond}$ and $\Xi_{QQ}^{*\diamond}$. The currents written down in Eq. (2) are taken in

the rest frame of hadrons. The corresponding expressions in a general frame moving with a velocity four-vector v^μ can be obtained by the substitution of $\boldsymbol{\gamma}^m \to \gamma_\perp^\mu = \gamma^\mu - \not{v} v^\mu$.

To compare with the full QCD analysis we represent the expression for the $J_{\Xi'^{\diamond}_{bc}}$ current given in [7]

$$J_{\Xi'^{\diamond}_{bc}} = \{r_1[u^{iT} C\gamma_5 c^j]b^k + r_2[u^{iT} C c^j]\gamma_5 b^k + r_3[u^{iT} C\gamma_5\gamma_m u c^j]\gamma^\mu b^k\}\varepsilon_{ijk},$$

so that the NRQCD structure can be obtained by the choice of $r_1 = r_2 = 1$ and $r_3 = 0$ and the antisymmetric permutation of c and b flavors. As we have already mentioned in the Introduction, the authors of [7] reported that the convergency of OPE is "bad" in the region around the point given by the NRQCD parameters. The instability results in huge uncertainties of estimates. To get rid this disadvantage, we analyze the NRQCD sum rules in details.

2.2 Description of the method

In this section we describe steps required for the evaluation of two-point correlation functions in the NRQCD approximation and the connection to physical characteristics of doubly heavy baryons. We start from the correlator of two baryonic currents with the half spin

$$\Pi(t) = i\int d^4x e^{ipx}\langle 0|T\{J(x), \bar{J}(0)\}|0\rangle = \not{v} F_1(t) + F_2(t), \tag{3}$$

where $t = k\cdot v$, k_μ and p_μ denote the residual and full momenta of doubly heavy baryon, respectively, v_μ is its four-velocity, which are related by the following formula:

$$p_\mu = k_\mu + (m_1 + m_2)v_\mu, \tag{4}$$

where m_1 and m_2 are the heavy quark masses. The appropriate definitions of scalar formfactors for the 3/2-spin baryon are given by the following:

$$\Pi_{\mu\nu}(t) = i\int d^4x e^{ipx}\langle 0|T\{J_\mu(x), \bar{J}_\nu(0)\}|0\rangle = -g_{\mu\nu}[\not{v}\tilde{F}_1(t) + \tilde{F}_2(t)] + \dots, \tag{5}$$

where we will not concern for contributions with distinct lorentz structures. The scalar correlators F can be evaluated in deep euclidean region by employing the Operator Product Expansion (OPE) in the framework of NRQCD for the time ordered product of currents in Eqs.(3), (5), say,

$$F_{1,2}(t) = \sum_d C_d^{(1,2)}(t) O_d, \tag{6}$$

where O_d denotes the local operator with a given dimension d: $O_0 = \hat{1}$, $O_3 = \langle\bar{q}q\rangle$, $O_4 = \langle\frac{\alpha_s}{\pi}G^2\rangle$, ..., and the functions $C_d(t)$ are the corresponding Wilson coefficients of OPE. In this work we include the nonperturbative contributions given by quark, gluon and mixed condensates. So, evaluating the contribution of quark condensate operator, we use the following OPE for the correlator of two quark fields [10]:

$$\langle 0|T\{q_i^a(x)\bar{q}_j^b(0)|0\rangle = -\frac{1}{12}\delta^{ab}\delta_{ij}\langle\bar{q}q\rangle\cdot[1 + \frac{m_0^2x^2}{16} + \frac{\pi^2x^4}{288}\langle\frac{\alpha_s}{\pi}G^2\rangle + ...], \tag{7}$$

where the value of mixed condensate is parameterized by introducing the variable m_0^2, which is numerically determined as $m_0^2 \approx 0.8$ GeV^2.

For the sake of simplicity, we write down the Wilson coefficients of unity and quark-gluon operators by making use of dispersion relations over t,

$$C_d(t) = \frac{1}{\pi}\int_0^\infty \frac{\rho_d(\omega)d\omega}{\omega - t}, \tag{8}$$

where ρ denotes the imaginary part in the physical region of NRQCD. Thus, the calculation of Wilson coefficients of operators under consideration is transformed into the evaluating the corresponding spectral densities.

To relate the NRQCD correlators to the real hadrons, we use the dispersion representation for the two point function with the physical spectral density, given by the appropriate resonance and continuum part. The coupling constants of baryons are defined by the following expressions:

$$\begin{aligned} \langle 0|J(x)|\Xi^{\diamond}_{QQ}(p)\rangle &= iZ_{\Xi^{\diamond}_{QQ}} u(v, M_\Xi) e^{ipx}, \\ \langle 0|J^m(x)|\Xi^{*\diamond}_{QQ}(p,\lambda)\rangle &= iZ_{\Xi^{*\diamond}_{QQ}} u^m(v, M_\Xi) e^{ipx}, \end{aligned}$$

where the spinor field with four-velocity v and mass M_Ξ satisfies the equation $\not{v}u(v, M_\Xi) = u(v, M_\Xi)$, and $u^m(v, M_\Xi)$ denotes the transversal spinor, so that $(\gamma^m - v^m\not{v})u^m(v, M_\Xi) = 0$.

We suppose that the continuum density, starting from the threshold ω_{cont} is equal to that of calculated in the framework of NRQCD. Then in sum rules equalizing the correlators, calculated in NRQCD and given by the physical states, the integrations above ω_{cont} cancel each other in two sides of relation. This fact leads to the dependence of calculated masses and couplings on the value of ω_{cond}. Further, we write down the correlators at the deep underthreshold point of $t_0 = -(m_1 + m_2) + t$ with $t \to 0$, which corresponds to the limit of $p^2 \to 0$. The approximation of single bound state leads to the following expression for the resonance contribution:

$$F_{1,2}(t) = \frac{M_\Xi |Z_\Xi|^2}{M_\Xi^2 - t^2}, \tag{9}$$

which can be expanded in series [1] over t. Then, the sum rules state the following equalities for the terms standing in front of powers of t

$$\frac{1}{\pi}\int_0^{\omega_{cont}} \rho_{1,2}(\omega)d\omega \frac{1}{(\omega + m_1 + m_2)^n} = |Z_\Xi|^2 \frac{1}{M_\Xi^n}, \tag{10}$$

where ρ_j contains the contributions given by various operators in OPE for the corresponding scalar correlators F_j. Introducing the following notation for n-th moment of two point correlation function

$$\mathcal{M}_n = \frac{1}{\pi}\int_0^{\omega_{cont}} \frac{\rho(\omega)d\omega}{(\omega + m_1 + m_2)^{n+1}}, \tag{11}$$

[1] To the nonrelativistic approximation we have to substitute for $\frac{1}{M_\Xi^2 - t^2}$ by the single pole in the physical region over t, $\frac{1}{(M_\Xi - t)(M_\Xi + t)} \approx \frac{1}{M_\Xi(M_\Xi - t)}$.

we can obtain the estimates of baryon mass M_Ξ, for example, as the following:

$$M_\Xi[n] = \frac{\mathcal{M}_n}{\mathcal{M}_{n+1}}, \tag{12}$$

and the coupling is determined by the expression

$$|Z_\Xi[n]|^2 = \mathcal{M}_n M_\Xi^{n+1}, \tag{13}$$

where we see the dependence of sum rule results on the scheme parameter. Therefore, we tend to find the region of parameter values, where, first, the result is stable under the variation of n, and, second, the both correlators F_1 and F_2 reproduce equal values of physical quantities: masses and couplings. The problem of full QCD was the second point: the difference between the evaluated variables.

The similar procedure can be described in the Borel scheme, wherein the consideration of $\mathcal{M}_n$ is replaced by the analysis of function $\mathcal{B}[w]$, which is defined by

$$\mathcal{B}[w] = \frac{1}{\pi}\int_0^{\omega_{cont}} \rho(\omega) d\omega e^{-(\omega+m_1+m_2)/w}, \tag{14}$$

equal to the resonance term given by

$$\mathcal{B}_\Xi[w] = |Z_\Xi|^2 e^{-M_\Xi/w}. \tag{15}$$

Then, again the scheme dependence of masses and couplings appears because of variation versus the Borel parameter w.

2.3 Calculating the spectral densities

In this subsection we present analytical expressions for the perturbative spectral functions in the NRQCD approximation. The evaluation of spectral densities involves the standard use of Cutkosky rules [11] with some modifications motivated by NRQCD. We explore the prescription that the discontinuity of two-point functions under consideration can be evaluated using the following substitutions for heavy and light quark propagators, correspondingly:

$$\begin{aligned}
\text{heavy quark}: &\quad \frac{1}{p_0 - (m + \frac{\vec{p}^2}{2m})} \to 2\pi i \cdot \delta(p_0 - (m + \frac{\vec{p}^2}{2m})),\\
\text{light quark}: &\quad \frac{1}{p^2 - m^2} \to 2\pi i \cdot \delta(p^2 - m^2).
\end{aligned}$$

For the perturbative spectral densities $\rho_{1,H}(\omega)$ and $\rho_{2,H}(\omega)$ standing in front of unity operator in F_1 and F_2, respectively, where $H = \Xi'^{\diamond}_{QQ'}, \Xi^{\diamond}_{QQ}$ or $\Xi^{*\diamond}_{QQ}$, we have the following expressions:

$$\begin{aligned}
\rho_{1,\Xi'^{\diamond}_{QQ'}}(\omega) &= \frac{1}{15015\pi^3(\omega+m_1+m_2)^3} 16\sqrt{2}\left(\frac{m_1 m_2}{m_1+m_2}\right)^{3/2} \omega^{7/2}[429m_1^3 + 429m_2^3 + \\
&\quad 715m_2^2\omega + 403m_2\omega^2 + 77\omega^3 + 143m_1^2(9m_2+5\omega) + 13m_1(99m_2^2 + \\
&\quad 110m_2\omega + 33\omega^2)], \quad (16)\\
\rho_{1,\Xi^{\diamond}_{QQ}}(\omega) &= 3\rho_{1,\Xi'^{\diamond}_{QQ'}}(\omega) = 3\rho_{1,\Xi^{*\diamond}_{QQ}}(\omega), \quad (17)\\
\rho_{2,\Xi^{\diamond}_{QQ}}(\omega) &= \rho_{2,\Xi'^{\diamond}_{QQ'}}(\omega) = \rho_{2,\Xi^{*\diamond}_{QQ}}(\omega) = 0. \quad (18)
\end{aligned}$$

Note that in the leading order of theory with the effective heavy quarks their spins are decoupled from the interaction, that causes the spin symmetry relations given in Eq.(17). The factor 3 stands because of the normalization of vector diquark current. We see also that in the leading approximation of perturbative NRQCD the F_2 correlators are equal to zero. This is because the interaction of massless light quark with the doubly heavy diquark is switched off in this order, and there is no mass term structure in the correlator.

The coulomb-like interaction inside the heavy diquark can be taken into account by the introduction of Sommerfeld factor $\mathbf{C}$ for the diquark spectral densities before the integration over the diquark invariant mass to obtain the baryon spectral densities, so that

$$\rho_{diquark} = \rho^B_{diquark} \cdot \mathbf{C}, \tag{19}$$

with

$$\mathbf{C} = \frac{2\pi\alpha_s}{3v_{12}}\left[1 - \exp\left(-\frac{2\pi\alpha_s}{3v_{12}}\right)\right]^{-1}, \tag{20}$$

where v_{12} denotes the relative velocity of heavy quarks inside the diquark, and we have taken into account the color anti-triplet structure of diquark. The relative velocity is given by the following expression:

$$v_{12} = \sqrt{1 - \frac{4m_1 m_2}{Q^2 - (m_1 - m_2)^2}}, \tag{21}$$

where Q^2 is the square of heavy diquark four-momentum. In NRQCD we take the limit of low velocities, so that denoting the diquark invariant mass squared by $Q^2 = (m_1 + m_2 + \epsilon)^2$ and the reduced quark pair mass by $m_{12} = m_1 m_2/(m_1 + m_2)$, we find

$$\mathbf{C} = \frac{2\pi\alpha_s}{3v_{12}}, \quad v_{12}^2 = \frac{\epsilon}{2m_{12}},$$

at $\epsilon \ll m_{12}$. The corrected spectral densities are equal to

$$\begin{aligned}
\rho^C_{1,\Xi'^{\diamond}_{QQ'}}(\omega) &= \frac{\alpha_s}{16\pi^2(\omega + m_1 + m_2)^3}\left(\frac{m_1 m_2}{m_1 + m_2}\right)^2 \omega^3[2m_1 + 2m_2^3 + \omega]^3, \\
\rho^C_{1,\Xi^{\diamond}_{QQ}}(\omega) &= 3\rho^C_{1,\Xi'^{\diamond}_{QQ'}}(\omega) = 3\rho^C_{1,\Xi^{*\diamond}_{QQ}}(\omega).
\end{aligned} \tag{22}$$

Further, the spectral functions, connected to the condensates of light quarks and gluons, can be derived. For the quark condensate term we have the following expressions:

$$\rho^{\bar{q}q}_{2,\Xi'^{\diamond}_{QQ'}}(\omega) = -\frac{\sqrt{2}}{\pi}\left(\frac{m_1 m_2}{m_1 + m_2}\right)^{3/2}\sqrt{\omega}, \tag{23}$$

$$\rho^{\bar{q}q}_{2,\Xi^{\diamond}_{QQ}}(\omega) = 3\rho^{\bar{q}q}_{2,\Xi'^{\diamond}_{QQ'}}(\omega) = 3\rho^{\bar{q}q}_{2,\Xi^{*\diamond}_{QQ}}(\omega), \tag{24}$$

$$\rho^{\bar{q}q}_{1,\Xi^{\diamond}_{QQ}}(\omega) = \rho^{\bar{q}q}_{1,\Xi'^{\diamond}_{QQ'}}(\omega) = \rho^{\bar{q}q}_{1,\Xi^{*\diamond}_{QQ}}(\omega) = 0, \tag{25}$$

which have to be multiplied by the Sommerfeld factor $\mathbf{C}$, wherein the variable ϵ is substituted by ω, since in this case we have no integration over the quark-diquark invariant mass.

It is interesting to stress that in NRQCD the light quark condensate contributes to the F_2 correlators, only. This fact has a simple physical explanation: to the leading order the

light quark operator can be factorized in the expression for the correlator of baryonic currents. Indeed, we can write down for the condensate contribution

$$\langle 0|T\{J(x),\bar{J}(0)\}|0\rangle = \langle 0|T\{q_i^a(x)\bar{q}_i^a(0)|0\rangle \cdot \frac{\hat{1}}{12}\cdot\langle 0|T\{J_d^j(x),\bar{J}_d^j(0)\}|0\rangle + \dots,$$

where $J_d^j(x)$ denotes the appropriate diquark current with the color index j, as it is defined by the baryon structure in Eqs. (2). So, we see that the restriction by the first term independent of x in the expansion for the quark correlator in (7) results in the independent contribution of diquark correlator to the baryonic one. Then, since the diquark correlator is isolated in F_2 from the baryonic formfactor F_1, the NRQCD sum rules lead to the evaluation of diquark mass and couplings from F_2, and estimation of baryon masses and couplings from F_1. These masses and couplings are different. The positive point is the possibility to calculate the binding energy for the doubly heavy baryons $\bar{\Lambda} = M_\Xi - M_{diquark}$. The disadvantage is the instability of NRQCD sum rules at this stage, since the various formfactors or correlators lead to the different results. In sum rules of full QCD various choices of parameters in the definitions of baryonic currents result in an admixture of pure diquark correlator in various formfactors, so that the estimations acquire huge uncertainties. Say, the characteristic ambiguity in the evaluation of baryon mass in full QCD is about 300 MeV, i.e. the value close to the expected estimate of $\bar{\Lambda}$. The analysis in the framework of NRQCD makes this result to be not unexpectable. Moreover, it is quite evident that the introduction of interactions between the light quark and the doubly heavy diquark destroys the factorization of diquark correlator. Indeed, we see that due to the higher terms in expansion (7), the diquark factorization is explicitly broken, which has to result in the convergency of estimates obtained from F_1 and F_2. Below we show numerically that this fact is valid. Technically, we point out that the contribution to the moments of spectral density, determined by the light quark condensate including the mixed condensate and the product of quark and gluon condensates, can be calculated after the exploration of (7), so that

$$\mathcal{M}_n^{q\bar{q}} = \left[1 - \frac{(n+2)!}{n!}\frac{m_0^2}{16} + \frac{(n+4)!}{n!}\frac{\pi^2}{288}\langle\frac{\alpha_s}{\pi}G^2\rangle\right]\ \mathcal{M}_n^{\langle\bar{q}q\rangle}. \tag{26}$$

For the corrections due to the gluon condensate we have

$$\begin{aligned}
\rho^{G^2}_{1,\Xi'^{\diamond}_{QQ'}}(\omega) &= \frac{\sqrt{\frac{m_1 m_2}{m_1+m_2}\omega}}{1344\sqrt{2}\pi(m_1+m_2)^2(\omega+m_1+m_2)^3}\cdot(28m_1^2+41m_1m_2+28m_2^2)\cdot \\
&\quad (\omega^3 - 7m_1^3 + 7m_1^2\omega - 21m_1^2m_2 + 7m_2\omega^2 + 7m_2^2\omega - 7m_2^3 + 7m_1\omega^2 \qquad (27)\\
&\quad +14m_1m_2\omega - 21m_1m_2^2),
\end{aligned}$$

$$\rho^{G^2}_{1,\Xi^{\diamond}_{QQ}}(\omega) = 3\rho^{G^2}_{1,\Xi'^{\diamond}_{QQ'}}(\omega) = 3\rho^{\bar{q}q}_{1,\Xi^{*\diamond}_{QQ}}(\omega), \tag{28}$$

$$\rho^{G^2}_{2,\Xi^{\diamond}_{QQ}}(\omega) = \rho^{G^2}_{2,\Xi'^{\diamond}_{QQ'}}(\omega) = \rho^{\bar{q}q}_{2,\Xi^{*\diamond}_{QQ}}(\omega) = 0, \tag{29}$$

which are written down for O_4, having the form $O_4 = \langle\frac{\alpha_s}{\pi}G^2\rangle$.

For the product of condensates $\langle\bar{q}q\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle$, wherein the gluon fields are connected to the heavy quarks in contrast to the light quark, we have computed the contribution to the two-point correlation function itself. It has the following form:

$$F^{\bar{q}qG^2}_{2,\Xi'^{\diamond}_{QQ'}}(\omega) = -\frac{\pi\sqrt{\frac{m_1m_2}{m_1+m_2}}(7m_1^2+8m_1m_2+7m_2^2)}{3072\sqrt{2}(-\omega)^{5/2}}, \tag{30}$$

$$F^{\bar{q}qG^2}_{2,\Xi^\diamond_{QQ}}(\omega) = 3F^{\bar{q}qG^2}_{2,\Xi'^\diamond_{QQ'}}(\omega) = 3F^{\bar{q}qG^2}_{2,\Xi^{*\diamond}_{QQ}}(\omega), \tag{31}$$

$$F^{\bar{q}qG^2}_{1,\Xi^\diamond_{QQ}}(\omega) = F^{\bar{q}qG^2}_{1,\Xi'^\diamond_{QQ'}}(\omega) = F^{\bar{q}qG^2}_{1,\Xi^{*\diamond}_{QQ}}(\omega) = 0, \tag{32}$$

where $\omega = -(m_1 + m_2) + t$ at the point under consideration, and the correlators have to be expanded in series of t, which gives the moments.

Thus, we provide the NRQCD sum rules, where we take into account the perturbative terms and the vacuum expectations of quark-gluon operators up to the contributions by the light quark condensate, gluon condensate, their product and the mixed condensate. Note, that the product of condensates is essential for the doubly heavy baryons, and we present the full NRQCD expression for this term, including the interaction of nonperturbative gluons with both the light and heavy quarks. The correct introduction of coulomb-like interactions is done for the perturbative spectral densities of heavy diquark, which is important for the nonrelativistic heavy quarks. Finally, we find the spin-symmetry relation for the baryon couplings in NRQCD

$$|Z_\Xi|^2 = 3|Z_{\Xi'}|^2 = 3|Z_{\Xi^*}|^2.$$

2.4 Anomalous dimensions for the baryonic currents

To connect the NRQCD sum rules to the quantities in full QCD we have to take into account the anomalous dimensions of effective baryonic currents with the nonrelativistic quarks. They determine the factors, which have to multiply the NRQCD correlators to obtain the values in full QCD. Indeed, to the leading order of NRQCD we have the relation

$$J^{QCD} = C_J(\alpha_s, \mu) \cdot J^{NRQCD},$$

where the coefficient C_J depends on the normalization scale μ and obeys the matching condition at the starting point of $\mu_0 = m_1 + m_2$. The anomalous dimensions of NRQCD currents are independent of the diquark spin structure in the leading order. They are equal to [8]

$$\begin{aligned}
\gamma &= \frac{d \ln C_J(\alpha_s, \mu)}{d \ln(\mu)} = \sum_{m=1}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^m \gamma^{(m)}, \\
\gamma^{(1)} &= \Big(-2C_B(3a-3) + 3C_F(a-2)\Big), \\
\gamma^{(2)} &= \frac{1}{6}(-48(-2+6\zeta(2))C_B^2 + C_A((104-240\zeta(2))C_B - 101C_F) \\
&\quad -64C_BN_FT_F + C_F(-9C_F + 52N_FT_F)),
\end{aligned} \tag{33}$$

where $C_F = (N_c^2-1)/2N_c$, $C_A = N_c$, $C_B = (N_c+1)/2N_c$, and $T_F = 1/2$ for $N_c = 3$, N_F being the number of light quarks. In Eq.(33) we give the one-loop result with the arbitrary gauge parameter a, and the two-loop anomalous dimension is represented in the Feynman gauge $a = 1$. So, numerically at $N_F = 3$ and $a = 1$ we find

$$\gamma^{(1)} = -4, \quad \gamma^{(2)} \approx -188.24. \tag{34}$$

In the leading logarithmic approximation and to the one-loop accuracy, the coefficient C_J is given by the expression

$$C_J(\mu) = \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{\frac{\gamma^{(1)}}{2\beta_0}} \tag{35}$$

where $\beta_0 = 11N_c/3 - 2N_F/3 = 9$. To evaluate the two-loop expression for C_J we have to know subleading corrections in the first α_s order in addition to the anomalous dimensions. These corrections are not available yet, so we restrict ourselves by the one-loop accuracy.

Further, we have to determine the normalization point for the NRQCD estimates $\mu = \mu_1$. We put it to the average momentum transfer inside the doubly heavy diquark, so that $\mu_1^2 = T_d 2m_{12}$, where T_d denotes the kinetic energy in the system of two heavy quarks, which is phenomenologically independent of the quark flavors and approximately equal to 0.2 GeV. Then, the coefficients C_J are equal to

$$C_{\Xi_{cc}} \approx 1.95, \quad C_{\Xi_{bc}} \approx 1.52, \quad C_{\Xi_{bb}} \approx 1.30, \tag{36}$$

with the characteristic uncertainty about 10% because of the variation of initial and final points $\mu_{0,1}$.

Finally, we emphasize that the values of C_J do not change the estimates of baryon masses calculated in the sum rules of NRQCD. However, they are essential in the evaluation of baryon couplings, which acquire these multiplicative factors.

3 Numerical estimates

In the present paper we explore the sum rule scheme of moments for the spectral functions of correlators. We have to stress that in this scheme the dominant uncertainty in the results is caused by the variation of heavy quark masses. In the analysis we chose the following region of mass values:

$$m_b = 4.6 - 4.7 \text{ GeV}, \quad m_c = 1.35 - 1.40 \text{ GeV}, \tag{37}$$

which is ordinary used in the sum rule estimates for the heavy quarkonia. Next critical point is the value of QCD coupling constant determining the coulomb-like interactions inside the doubly heavy diquark. Indeed, it stands linearly in front of the perturbative functions of diquark contributions. Thus, the introduction of α_s/v-corrections is essential for both the baryon couplings and the relative contributions of perturbative terms and condensates to the baryon masses. To decrease the uncertainty we impose the same approach to the heavy quarkonia, where it is well justified, and then, we extract the characteristic values for the heavy-heavy systems from the comparison of calculations with the current data on the leptonic constants of heavy quarkonia, which are known experimentally for $c\bar{c}$ and $b\bar{b}$ or evaluated in various approaches for $\bar{b}c$. So, our calculations give the following couplings of coulomb interactions

$$\alpha_s(b\bar{b}) = 0.37, \quad \alpha_s(c\bar{b}) = 0.45, \quad \alpha_s(c\bar{c}) = 0.60. \tag{38}$$

The dependence of estimates on the value of thershold for the continuum contribution is not so valuable as on the quark masses. We fix the region of ω_{cont} as

$$\omega_{cont} = 1.3 - 1.4 \text{ GeV}. \tag{39}$$

For the condensates of quarks and gluons the following regions are under consideration:

$$\langle \bar{q}q \rangle = -(250 - 270 \text{ MeV})^3, \quad m_0^2 = 0.75 - 0.85 \text{ GeV}^2, \quad \langle \frac{\alpha_s}{\pi} G^2 \rangle = (1.5 - 2) \cdot 10^{-2} \text{ GeV}^4. \tag{40}$$

So, we have described the choices of parameters.

Figs. 1-3 represent the calculated difference of masses extracted from the F_1 and F_2 correlators[2] for the baryons Ξ_{cc}, Ξ_{bc} and Ξ_{bb}. We certainly see that at low numbers of moments for the spectral densities, the baryon-diquark mass difference can be evaluated as

$$\bar{\Lambda} = 0.40 \pm 0.03 \text{ GeV}, \tag{41}$$

which is quite a reasonable value, being in a good agreement with the estimates in the heavy-light mesons. In the region of mass difference stability we can fix the number of moment for the spectral density, say, $n = 27 \pm 1$ for Ξ_{bc}, and calculate the corresponding masses of baryons, which are equal to

$$M_{\Xi_{cc}} = 3.47 \pm 0.05 \text{ GeV}, \quad M_{\Xi_{bc}} = 6.80 \pm 0.05 \text{ GeV}, \quad M_{\Xi_{bb}} = 10.07 \pm 0.09 \text{ GeV}, \tag{42}$$

where we do not take into account the spin-dependent splitting caused by the α_s-corrections to the heavy-light interactions, which are not available yet. The uncertainties in the mass values are basically given by the variation of heavy quark masses. The convergency of NRQCD sum rules allows one to improve the accuracy of estimates in comparison with the previous analysis in full QCD [7]. The obtained values are in agreement with the calculations in the framework of nonrelativistic potential models [5].

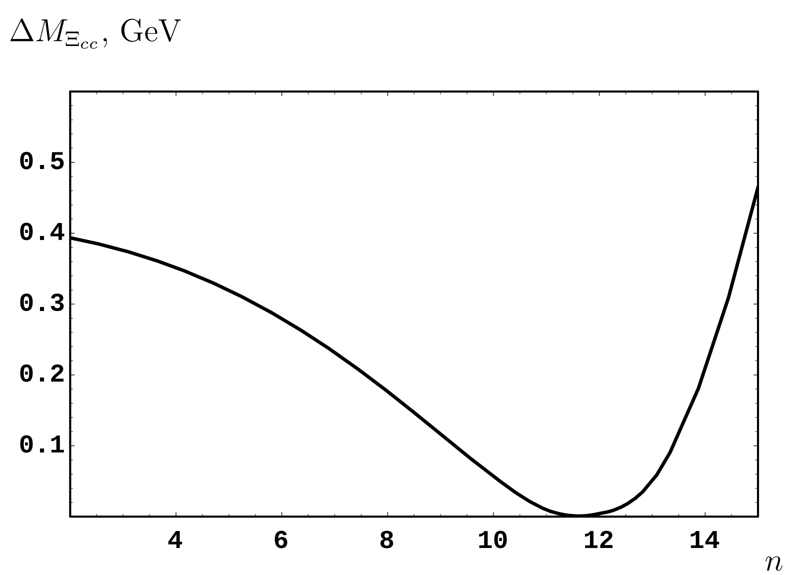

Figure 1: The difference between the Ξ_{cc}-baryon masses calculated in the NRQCD sum rules for the formfactors F_1 and F_2 in the scheme of moments for the spectral densities.

[2] In these figures we have fixed the value of gluon condensate $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 1.7 \cdot 10^{-2}$ GeV^4 and arranged m_0^2 in the above region to reach zero differences between the masses, though the variation of parameters leads to errors in the estimates quoted below.

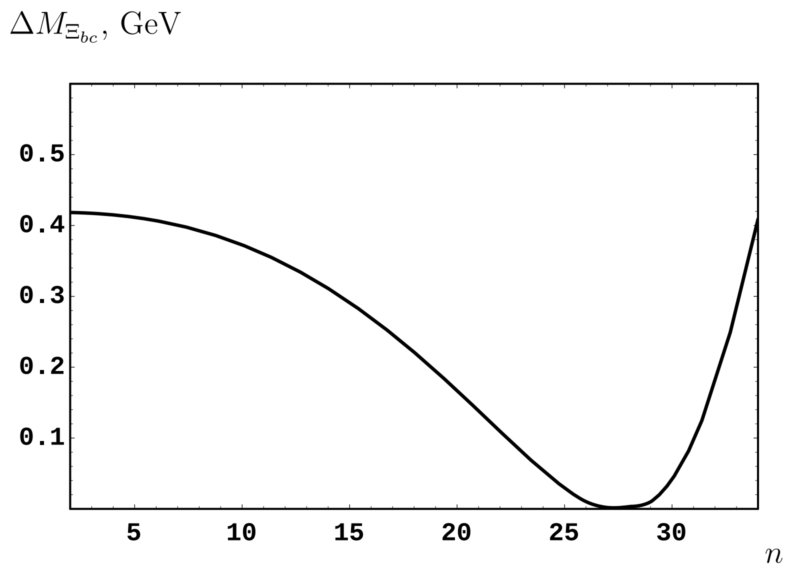

Figure 2: The difference between the Ξ_{bc}-baryon masses calculated in the NRQCD sum rules for the formfactors F_1 and F_2 in the scheme of moments for the spectral densities.

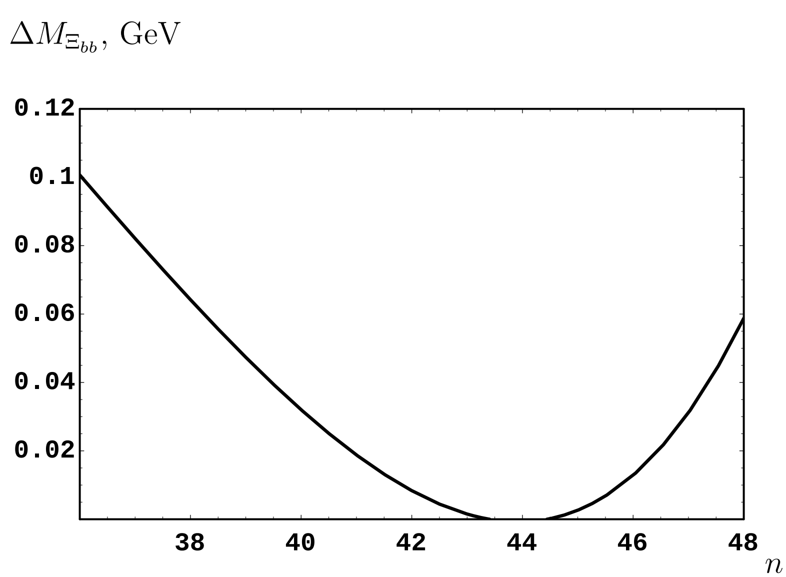

Figure 3: The difference between the Ξ_{bb}-baryon masses calculated in the NRQCD sum rules for the formfactors F_1 and F_2 in the scheme of moments for the spectral densities.

Figs. 4-6 show the dependence of baryon couplings calculated in the moment scheme of NRQCD sum rules. We find that the stability regions for $|Z_\Xi|^2$ determined from the F_1 and F_2 correlators coincide with those of the mass differences. So, the baryon couplings in NRQCD are equal to

$$\begin{aligned} |Z^{\rm NR}_{\Xi_{cc}}|^2 &= (1.7\pm 0.3)\cdot 10^{-3}\ {\rm GeV}^6, \\ |Z^{\rm NR}_{\Xi_{bc}}|^2 &= (3.7\pm 0.5)\cdot 10^{-3}\ {\rm GeV}^6, \\ |Z^{\rm NR}_{\Xi_{bb}}|^2 &= (1.5\pm 0.3)\cdot 10^{-2}\ {\rm GeV}^6. \end{aligned} \tag{43}$$

The values given above have to be multiplied by the Wilson coefficients coming from the expansion of full QCD operators in terms of NRQCD fields, as they have been estimated by use of corresponding anomalous dimensions. This procedure results in

$$\begin{aligned} |Z_{\Xi_{cc}}|^2 &= (6.5\pm 1.2)\cdot 10^{-3}\ {\rm GeV}^6, \\ |Z_{\Xi_{bc}}|^2 &= (8.5\pm 0.9)\cdot 10^{-3}\ {\rm GeV}^6, \\ |Z_{\Xi_{bb}}|^2 &= (2.5\pm 0.3)\cdot 10^{-2}\ {\rm GeV}^6. \end{aligned} \tag{44}$$

which are inside the regions given in the analysis of sum rules in full QCD.

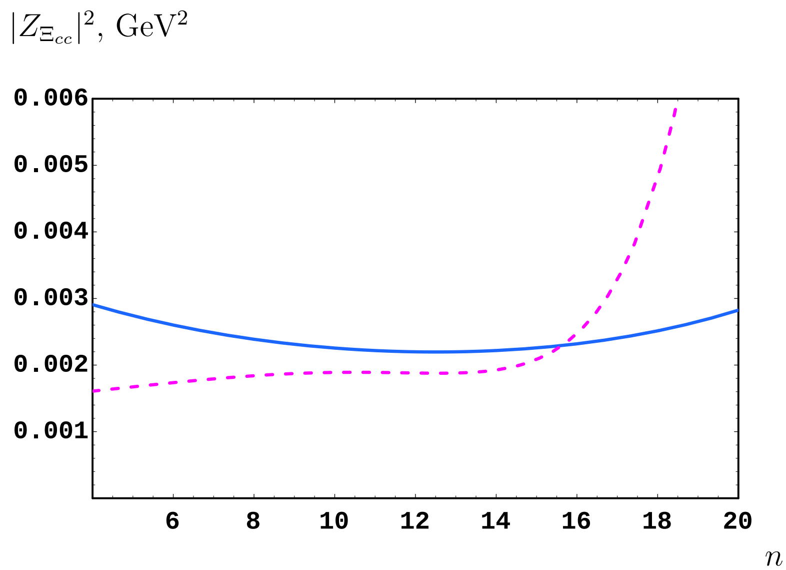

Figure 4: The couplings $|Z^{(1,2)}_{\Xi_{cc}}|^2$ of Ξ_{cc}-baryon calculated in the NRQCD sum rules for the formfactors F_1 and F_2 (solid and dashed lines, correspondingly) in the scheme of moments for the spectral densities.

For the sake of comparison, we derive the relation between the baryon coupling and the wave function of doubly heavy baryon evaluated in the framework of potential model in the approximation of quark-diquark factorization. So, we find

$$|Z^{\rm PM}| = 2\sqrt{3}|\Psi_d(0)\cdot\Psi_l(0)|, \tag{45}$$

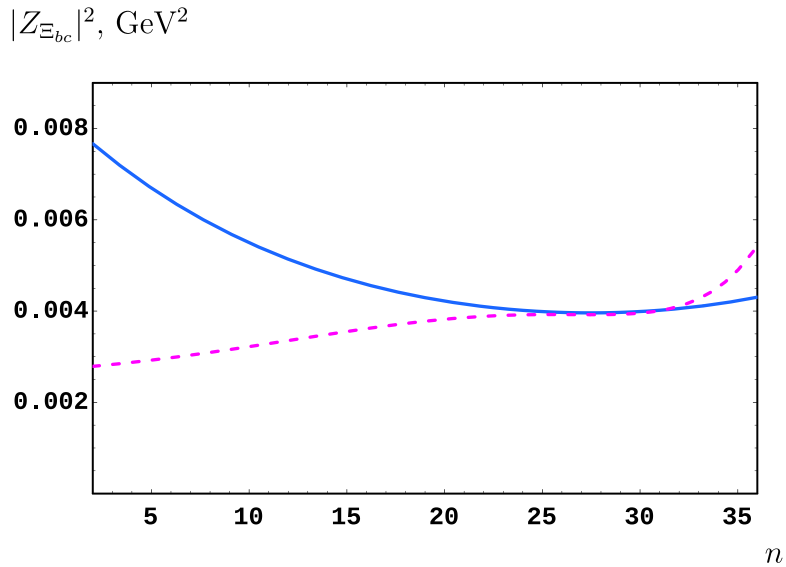

Figure 5: The couplings $|Z^{(1,2)}_{\Xi_{bc}}|^2$ of Ξ_{bc}-baryon calculated in the NRQCD sum rules for the formfactors F_1 and F_2 (solid and dashed lines, correspondingly) in the scheme of moments for the spectral densities.

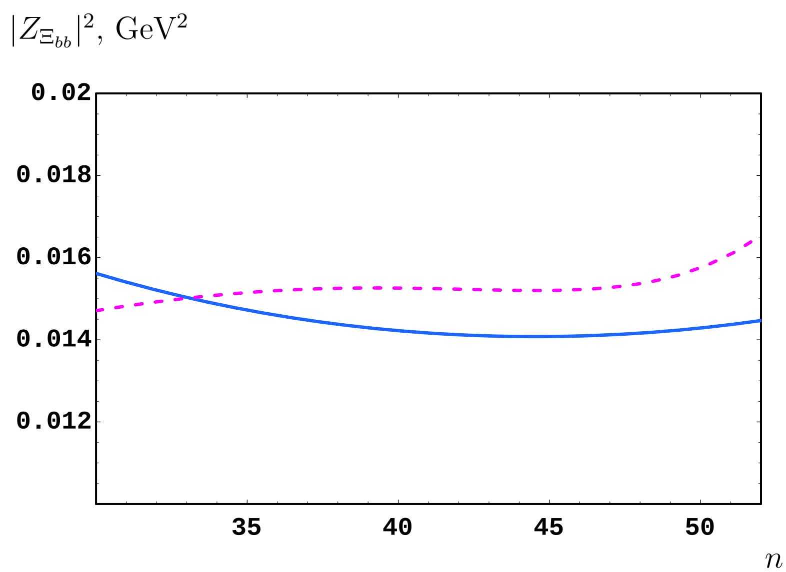

Figure 6: The couplings $|Z^{(1,2)}_{\Xi_{bb}}|^2$ of Ξ_{bb}-baryon calculated in the NRQCD sum rules for the formfactors F_1 and F_2 (solid and dashed lines, correspondingly) in the scheme of moments for the spectral densities.

where $\Psi_d(0)$ and $\Psi_l(0)$ denote the wave functions at the origin for the doubly heavy diquark and light quark-diquark systems, respectively. In the approximation used, the values of $\Psi(0)$ were calculated in [5] in the potential by Buchmüller–Tye [12], so that

$$\begin{aligned}
\sqrt{4\pi}\, |\Psi_l(0)| &= 0.53\ \mathrm{GeV}^{3/2},\\
\sqrt{4\pi}|\Psi_{cc}(0)| &= 0.53\ \mathrm{GeV}^{3/2},\\
\sqrt{4\pi}|\Psi_{bc}(0)| &= 0.73\ \mathrm{GeV}^{3/2},\\
\sqrt{4\pi}|\Psi_{bb}(0)| &= 1.35\ \mathrm{GeV}^{3/2}.
\end{aligned}$$

These parameters result in the estimates in the static limit of potential models

$$\begin{aligned}
|Z^{\mathrm{PM}}_{\Xi_{cc}}|^2 &= 6.0\cdot 10^{-3}\ \mathrm{GeV}^6,\\
|Z^{\mathrm{PM}}_{\Xi_{bc}}|^2 &= 1.1\cdot 10^{-2}\ \mathrm{GeV}^6,\\
|Z^{\mathrm{PM}}_{\Xi_{bb}}|^2 &= 3.9\cdot 10^{-2}\ \mathrm{GeV}^6.
\end{aligned} \tag{46}$$

As it is well known from the analysis of heavy-light mesons, the approximation of potential models for the static limit of coupling results in the overestimation of leptonic constants of heavy mesons with the single heavy quark because of the large corrections in the expansion of corresponding currents through the effective static fields. So, the contribution by the higher dimensional operators is essential, and it leads to the suppression factor of about 1/2 for the couplings. The introduction of this factor allows us to calculate the leptonic constants of heavy mesons with the same wave function of light quark

$$f_D = 185\ \mathrm{MeV}, \quad f_B = 115\ \mathrm{MeV},$$

which are quite reasonable estimates, being in agreement with the results of QCD sum rules [13]. Then, we use the same factor for the renormalization of baryon couplings and find

$$\begin{aligned}
|\bar{Z}^{\mathrm{PM}}_{\Xi_{cc}}|^2 &= 1.5\cdot 10^{-3}\ \mathrm{GeV}^6,\\
|\bar{Z}^{\mathrm{PM}}_{\Xi_{bc}}|^2 &= 2.8\cdot 10^{-3}\ \mathrm{GeV}^6,\\
|\bar{Z}^{\mathrm{PM}}_{\Xi_{bb}}|^2 &= 9.8\cdot 10^{-3}\ \mathrm{GeV}^6.
\end{aligned} \tag{47}$$

Comparing the values in (47) with ones in (43), we see a good agreement of NRQCD results on the baryon couplings with estimates of potential models improved by the correction factor, if we take into account the ordinary accuracy about 30% for the phenomenological quark models.

Finally, we suppose that the corrections coming from the higher orders of NRQCD expansion to the baryon couplings are not greater than 15%, since the diquark masses are quite large.

Thus, we obtain the reliable estimates of masses and couplings for the doubly heavy baryons in the framework of NRQCD sum rules.

4 Conclusion

We have considered the NRQCD sum rules for the two-point correlators of baryonic currents with two heavy quarks. The nonrelativistic approximation for the heavy quark fields allows one

to fix the structure of currents and to take into account the coulomb-like interactions inside the doubly heavy diquark. Moreover, we introduce the higher order operators responsible for the quark-gluon condensates to get the convergency of sum rule method for two scalar functions of correlators. To the leading approximation, including the perturbative term and the contributions of quark and gluon condensates, the correlators of three-quarks state and the doubly heavy diquark are factorized in separate functions, so that the sum rules result in the different values of masses and couplings. This fact indicates the divergency of approach unless the product of quark and gluon condensates and the mixed condensate are taken into account. Then, the interaction of two heavy quarks and light quark destroy the factorization, which allows one to get meaningful estimates of masses and couplings. Moreover, we have also calculated the binding energy of doubly heavy diquark, which is in a good agreement with the estimates in the framework of potential models.

Thus, the NRQCD sum rules allow us to improve the analysis of masses and couplings for the doubly heavy baryons and to obtain reliable results.

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References

[1] *F. Abe et al.*, CDF Collaboration, Phys. Rev. Lett. **81**, 2432 (1998), Phys. Rev. **D58**, 112004 (1998).

[2] *S.S.Gershtein, V.V.Kiselev, A.K.Likhoded, A.V.Tkabladze, A.V.Berezhnoy, A.I.Onishchenko*, Talk given at 4th International Workshop on Progress in Heavy Quark Physics, Rostock, Germany, 20-22 Sept. 1997, IHEP 98-22 [hep-ph/9803433];
S.S.Gershtein, V.V.Kiselev, A.K.Likhoded, A.V.Tkabladze, Phys. Usp. **38**, 1 (1995) [Usp. Fiz. Nauk **165**, 3 (1995)];
S.S.Gershtein et al., Phys. Rev. **D51**, 3613 (1995);
V.V.Kiselev, A.A.Likhoded, A.I.Onishchenko, preprint DESY 99-053 (1999) [hep-ph/9905359].

[3] *A.V.Berezhnoy, V.V.Kiselev, A.K.Likhoded, A.I.Onishchenko*, Phys. Rev. D57 (1997) 4385;
A.V.Berezhnoy, V.V.Kiselev, A.K.Likhoded, Z.Phys. **A356**, 89 (1996), Phys. Atom. Nucl. **59**, 870 (1996) [Yad. Fiz. **59**, 909 (1996)];
S.P.Baranov, Phys. Rev. D 56, 3046 (1997);
V.V.Kiselev, A.K. Likhoded, M.V. Shevlyagin, Phys. Lett. B332, 411 (1994);
A.Falk et al., Phys. Rev. D49, 555 (1994).

[4] *V.V.Kiselev, A.K.Likhoded, A.I.Onishchenko*, Phys. Rev. **D60**, 014007 (1999);
V.V.Kiselev, A.K.Likhoded, A.I.Onishchenko, preprint DESY 98-212 (1999) [hep-ph/9901224];
B.Guberina, B.Melic, H.Stefancic, Eur. Phys. J. **C9**, 213 (1999).

[5] *S.S.Gershtein, V.V.Kiselev, A.K.Likhoded, A.I.Onishchenko*, preprint IHEP 98-66 (1998) [hep-ph/9811212], Heavy Ion Phys **9**, 133 (1999), Mod. Phys. Lett. **A14**, 135 (1999);
D.Ebert, R.N.Faustov, V.O.Galkin, A.P.Martynenko, V.A.Saleev, Z. Phys. C76 (1997) 111;
J.G.Körner, M.Krämer, D.Pirjol, Prog. Part. Nucl. Phys. 33 (1994) 787;
R.Roncaglia, D.B.Lichtenberg, E.Predazzi, Phys. Rev. D52 (1995) 1722.

[6] *M.A.Shifman, A.I.Vainshtein, V.I.Zakharov*, Nucl. Phys. **B147**, 385 (1979);
L.J.Reinders, H.R.Rubinstein, S.Yazaki, Phys. Rep. **127**, 1 (1985).

[7] *E.Bagan, M.Chabab, S.Narison*, Phys. Lett. B306 (1993) 350;
E.Bagan at al., Z. Phys. **C64**, 57 (1994).

[8] *V.V.Kiselev, A.I.Onishchenko*, preprint IHEP 98-64 (1998) [hep-ph/9810283].

[9] *S. Groote, J.G. Körner and O.I. Yakovlev*, Phys. Rev. **D54**, 3447 (1996).

[10] *A.Smilga*, Yad. Phys. **35**, 473 (1982).

[11] *R.E.Cutkosky*, J. Math. Phys. **1**, 429 (1960).

[12] *W.Buchmüller and S.-H.H.Tye*, Phys. Rev. **D24**, 132 (1981).

[13] *S.Narison*, Phys. Lett. **B341**, 73 (1994), Phys. Lett. **B352**, 122 (1995).